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Effect of parallel transport currents on the d-wave Josephson junction

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Abstract
In this paper, the non-local mixing of coherent current states in d-wave superconducting banks is investigated. The superconducting banks are connected via a ballistic point contact. The banks have mis-orientation and phase difference. Furthermore, they are subjected to a tangential transport current along the *ab* plane of d-wave crystals and parallel to the interface between the superconductors. The effects of mis-orientation and external transport current on the current–phase relations and current distributions are the subjects of this paper. It is observed that, at values of phase difference close to 0, π and 2π, the current distribution may have a vortex-like form in the vicinity of the point contact. The current distribution of the above-mentioned junction between d-wave superconductors is totally different from the junction between s-wave superconductors. The interesting result which this study shows is that spontaneous and Josephson currents are observed for the case of $\phi = 0$.

1. Introduction

The weak link between two d-wave superconductors is a long-studied problem theoretically [1–9]. A theoretical investigation of the total transparent Josephson junction between two d-wave superconductors has been done in [1]. Using the quasi-classical approach, a d-wave Josephson Junction with low-transparent interface has been studied in [2]. Anisotropic and unconventional pairing symmetry has been considered for d–I–d systems and zero-energy states (ZES) have been considered as the result of sign change of the order parameter observed in paper [3]. A spontaneous current parallel to the interface between d-wave superconductors has been presented in paper [4]. The junction between current-carrying states of d-wave superconductors, has been investigated in [4]. The authors of paper [4] by numerical self-consistent calculations show that the supercurrent parallel to the junction may flow in the direction opposite to the current direction at the superconducting banks. The effect of the transparency of the d-wave Josephson junction interface has been studied in [5]. In [6], the effects of transparency and mis-orientation of two d-wave crystals have been investigated analytically. Using the Bogoliubov–de Gennes equations, the ZES as the origin of a zero bias conductance peak (ZBCP) have been studied in [7, 8]. ZES have been introduced as the fingerprint of unconventional pairing symmetry in [7, 8]. In [9], a special geometry of the d-wave superconducting layer as a weak link has been investigated and the π Josephson junction has been observed. Also, because of the high critical temperature of d-wave superconductors (and cheap production technology), much experimental work related to the d-wave weak link has been done in the last two decades [10–16]. A complete review of these experiments has been presented in a review paper [10]. A phase-sensitive experiment (phase of the superconducting order parameter) has been presented by the authors of [11] for the determination of the symmetry of the order parameter in high T_c cuprate superconductors. Using phase interference experiments in the Josephson junctions, d-wave pairing symmetry in the cuprate superconductors has been observed in [11–13]. A nonsinusoidal form of the current–phase diagram has been observed experimentally in [14]. The authors of [15] have measured the current–phase relationship of the symmetric grain boundary weak link and observed that the temperature controlled the sign change of the first harmonic of the Josephson current ($I(\phi) = I_1 \sin\phi + I_2 \sin 2\phi + \cdots$). Because of competition between the first and second harmonics of Josephson current, the nonmonotonic dependence of the critical current on temperature has been reported in [15]. An experimental investigation of a Josephson junction between two d-wave superconductors has been done and the effect of the insulator between them has been studied in [16]. They observed $0-\pi$ transitions by reducing the width of the insulator in the d–I–d Josephson junction. On the other hand, it is well known that non-locality and the Josephson effect coexist. Charged particles orbiting around a magnetic flux are influenced by it in the form of a phase difference, although the region including the flux is impenetrable for charged particles. This phase, as the Aharonov–Bohm phase, is a demonstration of the non-locality of quantum mechanics. While the supercurrent in a superconducting bulk depends on the phase gradient locally [17], $\mathbf{j}(\mathbf{r}) \propto \nabla\varphi(\mathbf{r})$, the Josephson supercurrent depends on phase difference non-locally [18], $\mathbf{j}(\varphi_2-\varphi_1) \propto \sin(\varphi_2-\varphi_1) \propto (\varphi_2-\varphi_1)$. The interplay between local supercurrent states and non-local phase difference between the superconducting bulks is an interesting problem. Local supercurrent states are introduced by the superfluid velocity of Cooper pairs. The non-locality of the Josephson current in the point contacts and the effect of superfluid velocity on the current states in narrow films and wires have been studied in [19]. An anomalous periodic behavior in terms of magnetic flux has been observed in [19]. This anomalous property is demonstrated as a result of a non-locality of supercurrents in the Josephson junction [19]. In addition, experimental results of [19] have been confirmed in analytical calculations of [20]. The dynamical Josephson junction between s-wave superconductors has been investigated in [21]. The authors of [21] studied the quantum interference between right and left s-wave superconductors, in which parallel transport currents flow. The existence of two antisymmetric vortex-like currents near the contact and at $\phi \simeq \pi$ as a new phenomenon was reported in [21]. The authors of [21] found that the total current is not the vector sum of the Josephson and transport currents because of a new term in the current. We called it the 'interference' current and it can also be named the 'parallel Josephson current'. In [21], the effect of reflection at the interface between s-wave superconductors has been investigated analytically and numerically.

In this paper, a planar weak link between two d-wave superconductors with a phase difference between their order parameters is investigated. The *ab* planes of two superconductors have a mis-orientation and the *c* axes of the two crystals are parallel to the interface between the d-wave superconductors. In the center of the interface I create an ideal transparent thin slit with length L and width a. Interference between wavefunctions of the left and right superconductors occurs through this slit. The remaining part of the interface is an ideal insulator and is impenetrable for Cooper pairs. Also, two transport supercurrents at the *ab* planes flow parallel to the insulator and the contact plane (see figure 1). The Josephson current from one of the bulks to another is a result of the interference between the states with phase difference ϕ, as is predicted in [18]. The contact (thin slit) scales, length L and width a, are larger than the Fermi wavelength and smaller than the coherence length of superconductivity. Furthermore, these scales are small compared to the mean free path of quasi-particles. Therefore the quasi-classical approximation for the ballistic point contact can be used. In this paper, the

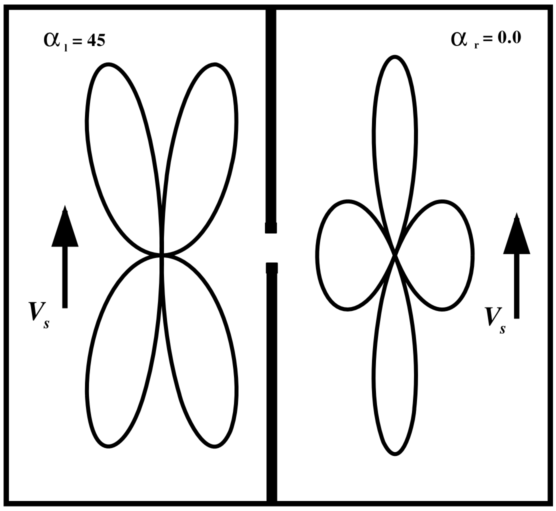

Figure 1. Model of the contact in the insulating partition, along the $\hat{\mathbf{y}}$, between two mis-oriented d-wave superconducting bulks with transport supercurrent on the banks. The plane of the paper is the *ab* plane of d-wave superconductors. In the d-wave superconductors like YBaCuO the *ab* plane is the plane of CuO.

Eilenberger equations for the above-mentioned structure, are solved and Green functions are obtained. The effects of mis-orientation and phase difference between order parameters and superfluid velocity on the current distributions and current–phase graphs are investigated.

The organization of the rest of this paper is as follows. In section 2 the quasi-classical equations for Green functions are presented. The obtained formulae for the Green functions are used to analyze a current state in the ballistic point contact. Also the effects of transport current and mis-orientation on the current distribution at the contact plane are investigated. In section 3 results of the simulation of the current distribution in the vicinity of the contact is investigated. An analytical investigation of this system near the critical temperature is in section 4. The paper's conclusions are presented in section 5.

2. Formalism and basic equations

The Eilenberger equations for the ξ-integrated Green's functions are used to describe the coherent current states in a superconducting ballistic micro-structure [23]:

$$\mathbf{v}_F \cdot \frac{\partial}{\partial \mathbf{r}} \widehat{G}_\omega(\mathbf{v}_F, \mathbf{r}) + [\omega\widehat{\tau}_3 + \widehat{\Delta}(\mathbf{v}_F, \mathbf{r}), \widehat{G}_\omega(\mathbf{v}_F, \mathbf{r})] = 0 \quad (1)$$

where

$$\widehat{\Delta} = \begin{pmatrix} 0 & \Delta \\ \Delta^\dagger & 0 \end{pmatrix}, \qquad \widehat{G}_\omega(\mathbf{v}_F, \mathbf{r}) = \begin{pmatrix} g_\omega & f_\omega \\ f_\omega^\dagger & -g_\omega \end{pmatrix} \quad (2)$$

Δ is the superconducting order parameter, $\widehat{\tau}_3$ is the Pauli matrix, and $\widehat{G}_\omega(\mathbf{v}_F, \mathbf{r})$ is the matrix Green function which depends on the electron velocity on the Fermi surface $\mathbf{v}_F$, the coordinate $\mathbf{r}$ and the Matsubara frequency $\omega = (2n+1)\pi T$, with n and T being an integer number and temperature, respectively. Also the normalization condition

$$g_\omega = \sqrt{1 - f_\omega f_\omega^\dagger} \tag{3}$$

with $f_\omega^\dagger$ being the time-reversal counterpart of f_ω should be satisfied by solutions of the Eilenberger equations. In general, Δ depends on the direction of $\mathbf{v}_\mathrm{F}$ and $\mathbf{r}$ and it can be determined by the self-consistent equation:

$$\Delta(\mathbf{v}_\mathrm{F}, \mathbf{r}) = 2\pi N(0) T \sum_{\omega>0} \langle V(\mathbf{v}_\mathrm{F}, \mathbf{v}'_\mathrm{F}) f_\omega(\mathbf{v}'_\mathrm{F}, \mathbf{r})\rangle_{\mathbf{v}'_\mathrm{F}} \tag{4}$$

and the current density by

$$\mathbf{j}(\mathbf{r}) = -4\pi \mathrm{i} e N(0) T \sum_{\omega} \langle \mathbf{v}_\mathrm{F} g_\omega(\mathbf{v}_\mathrm{F}, \mathbf{r})\rangle_{\mathbf{v}_\mathrm{F}} \tag{5}$$

respectively, where $V(\mathbf{v}_\mathrm{F}, \mathbf{v}'_\mathrm{F})$ is the interaction potential, $N(0)$ is the 2D density of states at the Fermi surface for each spin projection and $\langle\cdots\rangle$ is the averaging over directions of $\mathbf{v}_\mathrm{F}$. The solution of the matrix equation (1) together with the self-consistency gap equation (4), and the Maxwell equation for superfluid velocity and normalization condition determines the current $\mathbf{j}(\mathbf{r})$ in the system. The thicknesses of the d-wave superconductors are assumed to be smaller than the coherence length $\xi_0 = \frac{\hbar v_\mathrm{F}}{\pi\Delta}$. Thus the spatial distributions of $\Delta(\mathbf{r})$ and $\mathbf{j}(\mathbf{r})$ depend only on the coordinates in the plane of the film and the Eilenberger equation (1) reduces to 2D equations.

Also equation (1) for the Green functions $\widehat{G}_\omega(\mathbf{r}, \mathbf{v}_\mathrm{F})$ have to be supplemented by the condition of specular reflection at the region $(x = 0, |y| \geqslant a)$ and continuity of solutions across the point contact $(x = 0, |y| \leqslant a)$. Far from the contact the Green functions should be coincident with the bulk solutions and the current should be a homogeneous transport current along the y axis.

In this formalism, the current has to be determined by a self-consistent gap equation (4) together with the Maxwell equation for superfluid velocity (Ampere's law $\frac{\mathrm{d}^2 A_y(x)}{\mathrm{d}x^2} = -\mu_0 J_y^{\mathrm{tot}}(x)$ with μ_0 for free space and $v_\mathrm{s} = -\frac{e}{mc} A_y(x)$ for superfluid velocity as in paper [5, 24]). However, for simplicity, the self-consistency of the order parameter is ignored and a step function is considered for spatial dependence and I do not consider the effect of current distribution on the superfluid velocity. I believe that, as in the papers [25, 26], the self-consistent investigation of the d-wave Josephson junction does not show a qualitative difference from the non-self-consistent results.

For Δ and $\mathbf{v}_\mathrm{s}$ being constant at each half-plane an analytical solution for the Eilenberger equations can be found by the method of integration along the quasi-classical trajectories of quasi-particles. At any point, $\mathbf{r} = (x, y)$, all ballistic trajectories can be categorized as transit and non-transit trajectories.

For the transit trajectories the Green functions satisfy continuity at the contact and the non-transit trajectories satisfy the specular reflection condition at the partition, $(x = 0, |y| \geqslant a)$. Also, all transit and non-transit trajectories should satisfy the boundary conditions in the left and right bulks. Making use of the solutions of the Eilenberger equations, the following expression is obtained for the current at the slit:

$$\mathbf{j}(x = 0, |y| < a, \phi, \mathbf{v}_\mathrm{s}, \alpha_\mathrm{l}, \alpha_\mathrm{r}) = 4\pi e N(0) v_\mathrm{F} T \times \sum_{\omega>0} \left\langle \widehat{\mathbf{v}} \frac{\widetilde{\omega}(\Omega_\mathrm{l} + \Omega_\mathrm{r}) - \mathrm{i}\eta\Delta_\mathrm{l}\Delta_\mathrm{r}\sin\phi}{\Omega_\mathrm{l}\Omega_\mathrm{r} + \widetilde{\omega}^2 + \Delta_\mathrm{l}\Delta_\mathrm{r}\cos\phi} \right\rangle_{\widehat{\mathbf{v}}} \tag{6}$$

where $\Delta_\mathrm{l,r} = \Delta_0 \cos(2(\theta - \alpha_\mathrm{l,r}))$ for $\mathrm{d}_{x^2-y^2}$ symmetry $\Omega_\mathrm{l,r} = \sqrt{\widetilde{\omega}^2 + \Delta_\mathrm{l,r}^2}$, $\widetilde{\omega} = \omega + \mathrm{i}\mathbf{p}_\mathrm{F}\mathbf{v}_\mathrm{s}$ with ω being the Matsubara frequency, $\mathbf{v}_\mathrm{s}$ is the superfluid velocity and $\widehat{\mathbf{v}} = \mathbf{v}_\mathrm{F}/v_\mathrm{F}$ is the unit vector, $\eta = \mathrm{sgn}(v_x)$. In this non-stationary Josephson junction, $\mathbf{v}_\mathrm{s} \neq 0$, the current has both $\mathbf{j}_x$ and $\mathbf{j}_y$ components. I define the Josephson current, external transport current, spontaneous current and interference current as

$$\mathbf{j}_\mathrm{Joseph} = \mathbf{j}(\phi, \mathbf{v}_\mathrm{s}, \alpha_\mathrm{l}, \alpha_\mathrm{r})_{\hat{x}} \tag{7}$$

$$\mathbf{j}_\mathrm{Trans} = \mathbf{j}(\phi = 0, \mathbf{v}_\mathrm{s} \neq 0, \alpha_\mathrm{l} = \alpha_\mathrm{r} = 0)_{\hat{y}} = \mathbf{j}_\mathrm{Bulk}, \tag{8}$$

$$\mathbf{j}_\mathrm{Spont} = \mathbf{j}(\phi \neq 0, \mathbf{v}_\mathrm{s} = 0, \alpha_\mathrm{l} \neq 0, \alpha_\mathrm{r} \neq 0)_{\hat{y}} \tag{9}$$

$$\mathbf{j}_\mathrm{Inter} = \mathbf{j}(\phi, \mathbf{v}_\mathrm{s}, \alpha_\mathrm{l}, \alpha_\mathrm{r})_{\hat{y}} - \mathbf{j}_\mathrm{Spont} - \mathbf{j}_\mathrm{Trans} \tag{10}$$

respectively. The Josephson current, $\mathbf{j}_\mathrm{Joseph} = \mathbf{j}_x$, is normal to the interface between superconductors as was considered by Josephson in [18] and the parallel component of current $\mathbf{j}_y$ consists of the three above-mentioned current terms; an external transport current, spontaneous current and 'interference' current. The new current term, 'interference' current, depends on the superfluid velocity, the orientations (with respect to the interface) and the phase difference between order parameters. The current term, $\mathbf{j}_\mathrm{Inter}$, is completely different from the transport current term, $\mathbf{j}_\mathrm{Trans}$, on the banks.

Thus, in addition to the spontaneous current for the stationary d-wave Josephson junction (investigated in [6]) and the transport current, I observe another current parallel to the interface ($\mathbf{j}_\mathrm{Inter}$). In particular, at $\phi \simeq \pi$ it may go in the opposite direction to the external transport current on the banks (depending on the orientations). This sign reversal of tangential supercurrent, which is the origin of vortex-like currents near the orifice, has been observed already in relation to the d-wave junction in [4]. At the two sides of the vortex-like currents, the flowing currents are parallel and antiparallel to the external supercurrent at the superconducting banks. In [4] a superconductor–normal–superconductor trajectory for particles has been considered whereas in practice there is one superconductor coupled to the normal metal. So, because of one superconductor in the structure of [4] it was impossible to define the phase difference. In the case of the junction between s-wave superconductors in [21], it was observed that sign reversal can be seen only for $\phi \simeq \pi$. So, for the s-wave counterpart of the set-up of paper [4] it is impossible to see a sign reversal because phase difference ϕ has no meaning for a system including one superconductor. However, in our calculations in this paper I have observed that this sign reversal and vortex appearance can be seen at $\phi = 0$, depending on mis-orientation. In the case of the d-wave in paper [4] in the absence of a phase difference, a sign reversal of current has been observed. I also observed a sign change of current and vortex for suitable mis-orientation of ab planes ($\alpha_\mathrm{l} = 0, \alpha_\mathrm{r} = \frac{\pi}{4}$), which confirms the results of paper [4]. The planes $\alpha_\mathrm{l} = 0$ and $\alpha_\mathrm{r} = \frac{\pi}{4}$ correspond to the planes (100) and (111), respectively.

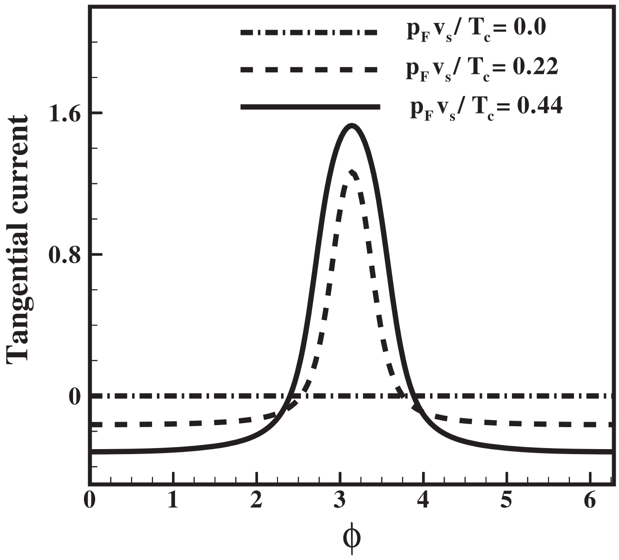

Figure 2. Tangential current j_y versus ϕ for $T/T_c = 0.1$, $\alpha_l = \alpha_r = 0$ in units of $j_0 = 4\pi e N(0) v_F T$. This is like the case of a junction between conventional superconductors [21].

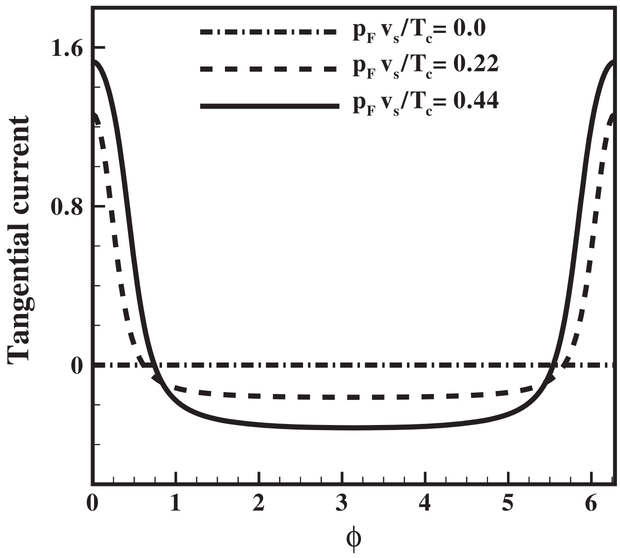

Figure 3. Tangential current j_y versus phase ϕ for $T/T_c = 0.1$, $\alpha_l = 0$ and $\alpha_r = \frac{\pi}{2}$.

3. Discussion

I have investigated the effects of mis-orientation and superfluid velocity on the current–phase relation and current distribution numerically and the results are as follows.

(1) The vortex-like currents appear at values of ϕ when the parallel current is maximum and positive while the external transport current is negative (figures 2–5).

(2) At $\alpha_l = \alpha_r = 0$ and arbitrary ϕ, the current distributions and current–phase graphs are identical with the s-wave which is investigated in [21]. For example, at $\alpha_l = \alpha_r = 0$ and $\phi = \pi$ two anti-symmetric vortex-like currents are observed and their common axis is normal to the interface (figure 6).

(3) At $\alpha_l = 0$, $\alpha_r = \pm\frac{\pi}{4}$ and $\phi = \pi$, two vortex-like currents are observed. Their common axis is rotated as much as $\mp\frac{\pi}{4}$ (figures 8 and 9). This case occurs for $\alpha_l = 0$, $\phi = \pi$ and arbitrary α_r with the rotated axis rotating as much as $(-\alpha_r)$.

(4) The appearance of vortex-like currents can be controlled by mis-orientation. For example, at $\phi = 0$ and $\alpha_l = \alpha_r = 0$, I cannot observe the vortex-like currents in the same manner that I cannot observe in the s-wave junction (figure 7). However, for $\phi = 0$, $\alpha_{l,r} = 0$, $\alpha_{r,l} = \frac{\pi}{2}$ the vortex-like currents appear, since, from (6) and for $d_{x^2-y^2}$ symmetry, I have

$$\mathbf{j}(\mathbf{r}, \phi = \pi, \mathbf{v}_s, \alpha_l, \alpha_r) = \mathbf{j}\left(\mathbf{r}, \phi = 0, \mathbf{v}_s, \alpha_l, \alpha_r + \frac{\pi}{2}\right). \tag{11}$$

(5) In figures 3–5 it is observed that, for $\phi = 0$ and consequently without any external magnetic flux, the interference between coherent current states can occur and the vortex-like currents can be observed because mis-orientation plays the role of the phase difference in equation (11).

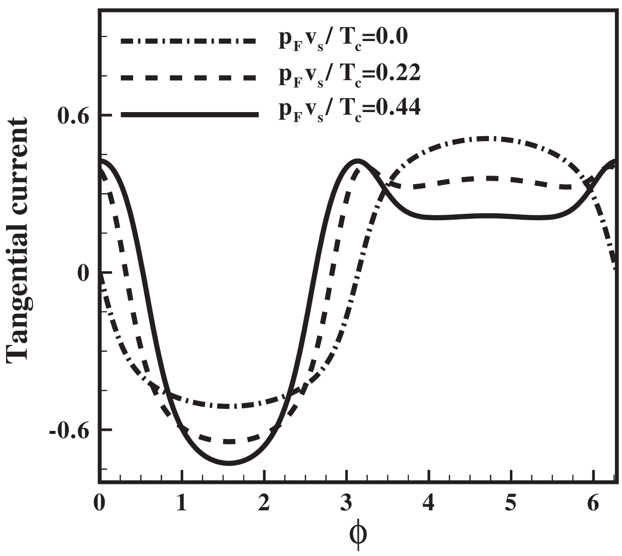

Figure 4. Tangential current j_y versus phase ϕ for $T/T_c = 0.1$, $\alpha_l = 0$ and $\alpha_r = \frac{\pi}{4}$.

(6) The parallel current, $\mathbf{j}_y$, is plotted in terms of the phase difference for different superfluid velocities and at $\alpha_l = \alpha_r = 0$, the maximum value of current and the appearance of vortex-like currents occur at $\phi = \pi$. In this case, far from $\phi = \pi$, I observe a constant current that is the external transport current on the banks (figure 2).

(7) For $\alpha_{l,r} = 0$, $\alpha_{r,l} = \pm\frac{\pi}{4}$ the maximum values of the parallel current, $\mathbf{j}_y$, and consequently the vortices, appear at $\phi = 0$, $\phi = \pi$ and $\phi = 2\pi$ (figures 4 and 5).

(8) For $\alpha_{l,r} = 0$, $\alpha_{r,l} = \frac{\pi}{2}$ the current–phase graphs are similar to the case of $\alpha_l = \alpha_r = 0$ but a displacement as much as π occurs (figures 2 and 3). Thus the vortices can be observed at $\phi = 0$ and $\phi = 2\pi$. But at $\phi = \pi$ I do not observe the vortex-like currents. This can be another difference between conventional and unconventional Josephson junctions.

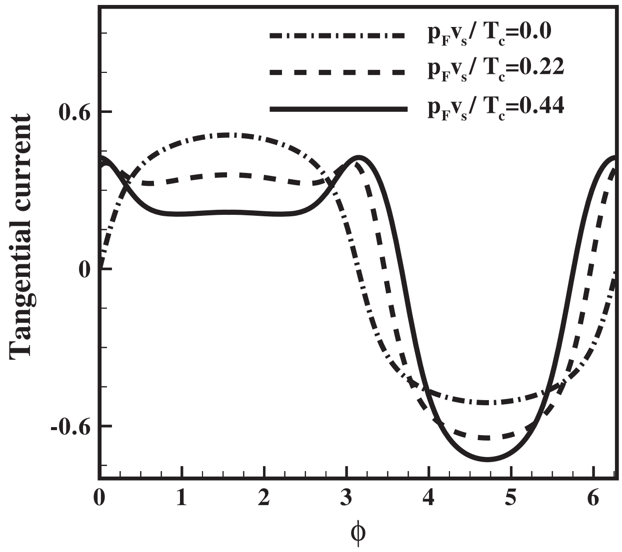

Figure 5. Tangential current j_y versus phase ϕ for $T/T_c = 0.1$, $\alpha_l = 0$ and $\alpha_r = -\frac{\pi}{4}$.

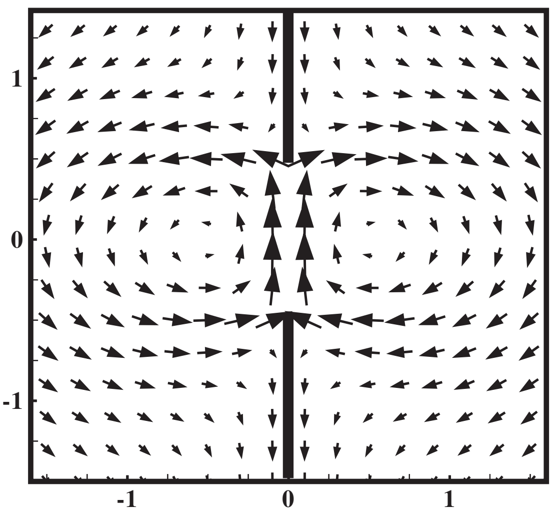

Figure 6. Vector plot of the current for $T/T_c = 0.1$, $P_F v_s/\Delta_0(0) = 0.5$, $\phi = \pi$, $\alpha_l = \alpha_r = 0$. Axes are marked in units of a. Because of our non-self-consistent formalism $\xi_0 = 5a$. It is similar to the s-wave junction in [21, 22].

(9) The superposition of dashed lines in figures 4 and 5 for zero superfluid velocity, and all the lines of figure 2, for zero mis-orientations apparently give us the three lines of figures 4 and 5. This means that in this case the tangential current is a superposition of transport current, 'interference' current [21] and spontaneous current [6].

(10) The tangential current for $\alpha_l = \alpha_r = 0$ and $\alpha_{l,r} = 0, \alpha_{r,l} = \frac{\pi}{2}$ is an even function of Josephson phase ϕ but for $\alpha_{l,r} = 0, \alpha_{r,l} = \pm\frac{\pi}{4}$ it is neither an even nor an odd function of the phase difference ϕ and the symmetry will be broken.

The superfluid of pairs creates the transport current but the spontaneous current is produced by both. mis-orientation and phase difference However, the 'interference' current depends

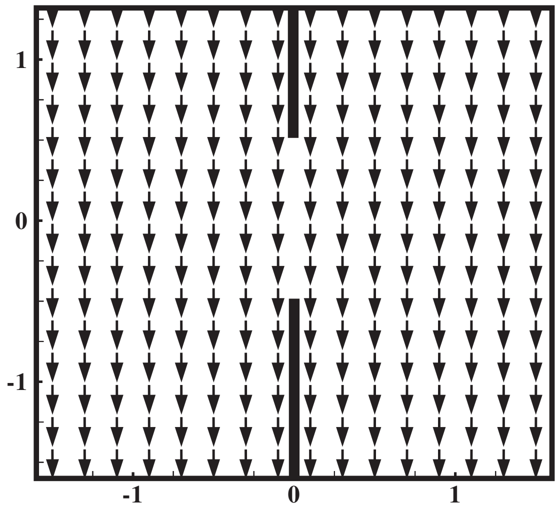

Figure 7. Vector plot of the current for $\phi = 0$, $\alpha_l = 0$, $\alpha_r = 0$ and $T/T_c = 0.1$, $P_F v_s/\Delta_0(0) = 0.5$. Vortices disappear but transport supercurrent flows.

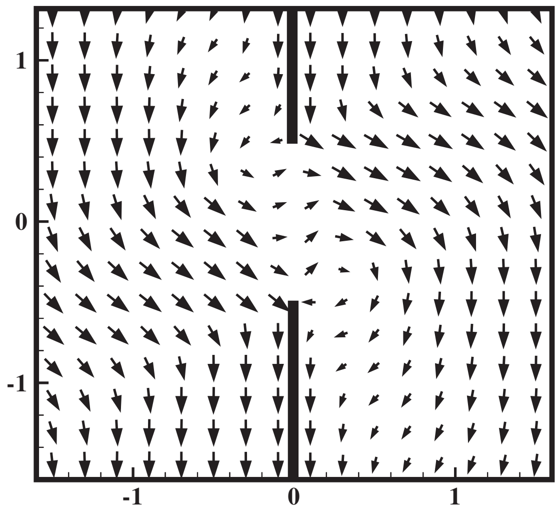

Figure 8. Vector plot of the current for $\phi = \pi$, $\alpha_l = 0$, $\alpha_r = \frac{\pi}{4}$ and $T/T_c = 0.1$, $P_F v_s/\Delta_0(0) = 0.5$. Axes are marked in units of a.

on all of the parameters (phase difference, mis-orientation and superfluid velocity) and can be the result of the non-locality of the supercurrent.

The spatial distributions of the order parameter and the current near and precisely at the contact are calculated using the Green function along the transit and non-transit trajectories numerically. Transit trajectories for each point come from the orifice (transparent part of the interface $x = 0, |y| \leqslant a$) while non-transit trajectories form the remaining part of the interface which is impenetrable (reflective part of the interface $x = 0, |y| \geqslant a$). The current distributions are calculated and simulated numerically for $T/T_c = 0.1$, $p_F v_s/\Delta_0(0) = 0.5$ and various choices of mis-orientation for $\phi = \pi$ (figures 8, 6 and 9) and $\phi = 0$ (figure 7). The 'interference' current that plays a central role for the production of vortex-like current has been observed here and also in [21]. At the phase values $0 < \phi < \frac{\pi}{2}$, $\frac{3}{2}\pi < \phi < 2\pi$ and $\alpha_l = \alpha_r = 0$, the interference

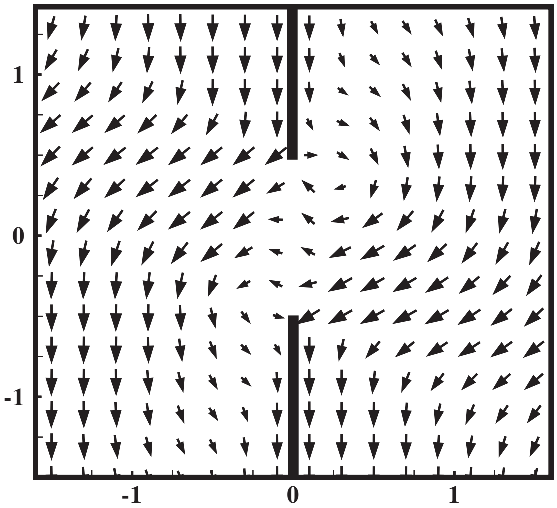

Figure 9. Vector plot of the current for $\phi = \pi$, $\alpha_l = 0$, $\alpha_r = -\frac{\pi}{4}$ and $T/T_c = 0.1$, $P_F v_s/\Delta_0(0) = 0.5$. The axis of vortices is rotated.

current is very small and thus the total current is close to the vector sum of transport, spontaneous and Josephson currents. But, for $\frac{\pi}{2} < \phi < \frac{3}{2}\pi$ the 'interference' current appears and the total current deviates from the vector sum of the three old currents [21]. Also, for zero mis-orientation the spontaneous current is zero.

The 'interference' current is always antiparallel to the transport current. But the spontaneous current may be parallel or antiparallel to the transport current. If the algebraic sum of transport, 'interference' and spontaneous currents is antiparallel to the transport current on the banks, the vortex-like currents can be observed (figure 6).

Thus the appearance of the vortex-like currents can be controlled by mis-orientation and phase difference. It is remarkable that, far from the contact $|\mathbf{r}| \sim \xi_0 > a$ for all ϕs, mis-orientations, temperatures T and superfluid velocities, the distributions of currents tend to the tangential transport currents on the banks.

4. Near the critical temperature

For temperatures close to the critical temperature, $T_c - T \ll T_c$, the problem can be solved analytically. At the contact I have

$$\mathbf{j} = \mathbf{j}_{\text{Joseph}} + \mathbf{j}_{\text{Spont}} + \mathbf{j}_{\text{Trans}} + \mathbf{j}_{\text{Inter}} \tag{12}$$

$$\mathbf{j}_{\text{Joseph}} = 2 j_c \sin\phi \left\langle \widehat{\mathbf{v}}_x \,\text{sgn}(v_x) \left(\frac{\Delta_l \Delta_r}{\Delta_0^2}\right)\right\rangle_{\widehat{\mathbf{v}}} \tag{13}$$

$$\mathbf{j}_{\text{Spont}} = 2 j_c \sin\phi \left\langle \widehat{\mathbf{v}}_y \,\text{sgn}(v_x) \left(\frac{\Delta_l \Delta_r}{\Delta_0^2}\right)\right\rangle_{\widehat{\mathbf{v}}} \tag{14}$$

$$\mathbf{j}_{\text{Trans}} = -j_c k \left\langle \widehat{\mathbf{v}} \widehat{v}_y \left(\frac{\Delta_l \Delta_r}{\Delta_0^2}\right)\right\rangle_{\widehat{\mathbf{v}}} \tag{15}$$

$$\mathbf{j}_{\text{Inter}} = j_c k (1 - \cos\phi) \left\langle \widehat{\mathbf{v}} \widehat{v}_y \left(\frac{\Delta_l \Delta_r}{\Delta_0^2}\right)\right\rangle_{\widehat{\mathbf{v}}} \tag{16}$$

where, as in [21]

$$j_c(T, v_s) = \frac{\pi |e| N(0) v_F}{8} \frac{\Delta_0{}^2(T, v_s)}{T_c} \tag{17}$$

is a critical current of the contact at $(T_c - T) \ll T_c$, k is a standard notation:

$$k = (14\varsigma(3)/\pi^3)(v_s p_F/T_c). \tag{18}$$

For the high value of temperatures near the T_c, the critical values of currents have a linear dependence on the $\Delta_0{}^2$, which can be replaced by $\Delta_0 = \sqrt{(\frac{32\pi^2}{21\zeta(3)})T_c(T_c - T)}$. On the other hand, the spontaneous and Josephson currents are the sinusoidal functions of the phase difference, as is expected for the currents near the T_c. Contrary to these two current terms, the 'interference' current is an even function of the phase difference. The current is divided into four parts, Josephson current $\mathbf{j}_{\text{Joseph}}$, spontaneous current, transport current in the banks $\mathbf{j}_{\text{Trans}}$ and the 'interference' current $\mathbf{j}_{\text{Inter}}$. It is observed that the currents generally and near the T_c obviously, depend not only on the mis-orientation $|\alpha_l - \alpha_r|$ but also on the orientations with respect to the interface. Because in the expressions $\langle \widehat{\mathbf{v}} \Delta_l \Delta_r \cdots \rangle$, the result of angular integrations may include both $|\alpha_l - \alpha_r|$ and $|\alpha_l + \alpha_r|$ terms. For example, for Josephson and spontaneous currents in (13) and (14) by angular integration on the Fermi surface I have

$$\mathbf{j}_{\text{Joseph}} = \left(\frac{2 j_c \sin\phi}{15\pi}\right)[15\cos(2\alpha_l - 2\alpha_r) - \cos(2\alpha_l + 2\alpha_r)]\widehat{\mathbf{x}} \tag{19}$$

and for the spontaneous current

$$\mathbf{j}_{\text{Spont}} = \left(\frac{-8 j_c \sin\phi}{15\pi}\right)\sin(2\alpha_l + 2\alpha_r)\widehat{\mathbf{y}}. \tag{20}$$

Also it is found that, for $\phi = \pi$ and exactly at the contact, the 'interference' current $\mathbf{j}_{\text{Inter}}$ is antiparallel to the $\mathbf{j}_{\text{Trans}}$. For $\phi = \pi$ the 'interference', Josephson and spontaneous currents are $\mathbf{j}_{\text{Inter}} = -2\mathbf{j}_{\text{Trans}}$, $\mathbf{j}_{\text{Joseph}} = 0$ and $\mathbf{j}_{\text{Spont}} = 0$, respectively, Consequently it is found that $\mathbf{j}_y = \mathbf{j}_{\text{Trans}} + \mathbf{j}_{\text{Inter}} + \mathbf{j}_{\text{Spont}} = -\mathbf{j}_{\text{Trans}}$. In this case while the Josephson current is zero the terms $\mathbf{j}_y$ and $\mathbf{j}_{\text{Trans}}$ that are directed opposite to each other control the appearance of vortex-like currents in the vicinity of the point contact. In addition, for $\phi = 0$, $\alpha_{l,r} = 0$ and $\alpha_{r,l} = \frac{\pi}{2}$, I can observe the vortex-like currents. This property can be a difference between d-wave and s-wave Josephson junctions, because in the s-wave Josephson junction the vortex-like currents are observed only for $\phi = \pi$ [21], while in the present calculations for d-wave Josephson junctions, the vortex-like currents may appear even for $\phi = 0$. Thus, for fixed values of the temperature and superfluid velocity, the presence of vortex-like currents can be controlled by mis-orientation and phase difference.

5. Conclusions

I observed the vortex-like currents in the vicinity of the point contact for d-wave Josephson junctions as well as for s-wave junctions in [21]. The interference current as a result of non-local supercurrent states appears. It may flow opposite to the external transport current. For $\phi = \pi$, $\alpha_l = 0$ and $\alpha_r = \pm\frac{\pi}{4}$ vortex-like currents with the rotated axis are observed. But, as is obtained in [21], the axis of vortex-like currents in the s-wave Josephson junction is normal to the interface. Thus, this rotated axis can be used to distinguish between s-wave and d-wave junctions. Also it can be exerted to distinguish between the junction between two pure s-waves and the junction between the mixing of conventional and unconventional order parameters (e.g. $d + \mathrm{i}s$). In addition to the ZES, this rotated axis can be another 'fingerprint' of d-wave pairing symmetry. Another interesting result is the behavior of the system in the absence of phase difference. For the s-wave system only in the presence of the phase difference, $\phi \simeq \pi \neq 0$, the vortex-like currents appear [21], while for the d-wave Josephson junction at zero phase and zero external magnetic flux, it is possible to observe the vortex-like currents for some mis-orientations. This can be a theoretical reason that the mis-orientation plays a role instead of the phase difference (Josephson phase). In the stationary Josephson junction $\mathbf{v}_s = \mathbf{0}$ the tangential interference current (spontaneous current) will be observed only for $\phi \neq 0$ but in the opposite case $\mathbf{v}_s \neq \mathbf{0}$ the tangential current, even in the absence of phase difference, may be observed. In addition, this tangential current can flow in the opposite direction to the external transport current and this factor can produce the vortex-like currents. Finally, the mis-orientation of superconducting *ab* planes (pairing symmetry in the momentum space) playing the role of the magnetic Josephson phase ($\phi = \frac{q\Phi}{\hbar}$, where q and Φ are electric charge and magnetic flux, respectively) is a reason for the magnetic nature of the pairing mechanism in high T_c superconductors which remains as an unknown and famous problem.

References

[1] Yip S 1995 *Phys. Rev.* B **52** 3087
[2] Barash Yu S, Galaktionov A V and Zaikin A D 1995 *Phys. Rev.* B **52** 665
[3] Tanaka Y and Kashiwaya S 1997 *Phys. Rev.* B **56** 892
[4] Fogelström M, Rainer D and Sauls J A 1997 *Phys. Rev. Lett.* **79** 281
[5] Fogelström M, Yip S and Kurkijärvi J 1998 *Physica* C **294** 289
[6] Rashedi G 2008 *J. Phys.: Conf. Ser.* **97** 012334
[7] Lofwander T, Shumeiko V S and Wendin G 2000 *Phys. Rev.* B **62** R14653
[8] Lofwander T, Shumeiko V S and Wendin G 2001 *Supercond. Sci. Technol.* **14** R53
[9] Gumann A, Iniotakis C and Schopohl N 2007 *Appl. Phys. Lett.* **91** 192502
[10] Golubov A A, Kupriyanov M Yu and Il'ichev E 2004 *Rev. Mod. Phys.* **76** 411
[11] Van Harlingen D J 1995 *Rev. Mod. Phys.* **67** 515
[12] Tsuei C C and Kirtley J R 2000 *Phys. Rev. Lett.* **85** 182
[13] Il'ichev E, Zakosarenko V, IJsselsteijn R P J, Hoenig H E, Schultze V, Meyer H-G, Grajcar M and Hlubina R 1999 *Phys. Rev.* B **60** 3096
[14] Il'ichev E, Zakosarenko V, IJsselsteijn R P J, Hoenig H E, Meyer H-G, Fistul M V and Müller P 1999 *Phys. Rev.* B **59** 11502
[15] Il'ichev E, Zakosarenko V, IJsselsteijn R P, Schultze V, Meyer H-G, Hoenig H E, Hilgenkamp H and Mannhart J 1998 *Phys. Rev. Lett.* **81** 894
[16] Testa G, Sarnelli E, Monaco A, Esposito E, Ejrnaes M, Kang D-J, Mennema S H, Tarte E J and Blamire M G 2005 *Phys. Rev.* B **71** 134520
[17] Tinkham M 1996 *Introduction to Superconductivity* 2nd edn (Singapore: McGraw-Hill)
[18] Josephson B D 1962 *Phys. Lett.* **1** 251
[19] Heida J P, Van Wees B J, Klapwijk T M and Borghs G 1998 *Phys. Rev.* B **57** R5618
[20] Lederman U, Fauchere A L and Blatter G 1999 *Phys. Rev.* B **59** R9027
[21] Rashedi G and Kolesnichenko Yu A 2004 *Phys. Rev.* B **69** 024516
[22] Kolesnichenko Yu A, Omelyanchouk A N and Shevchenko S N 2003 *Phys. Rev.* B **67** 172504
[23] Eilenberger G 1968 *Z. Phys.* **214** 195
[24] Krawiec M, Gyorffy B L and Annett J F 2004 *Phys. Rev.* B **70** 134519
[25] Amin M H S, Omelyanchouk A N and Zagoskin A M 2001 *Phys. Rev.* B **63** 212502
[26] Amin M H S, Omelyanchouk A N, Rashkeev S N, Coury M and Zagoskin A M 2002 *Physica* B **318** 162